

A Survey of Cellular Automata: Types, Dynamics, Non-uniformity and Applications

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Cellular automata (CAs) are dynamic frameworks which exhibit complex global behavior from simple local interaction and computation. Since the inception of CA by von Neumann in 1950s, it has attracted the attention of several researchers over various backgrounds and fields for modeling different physical, natural as well as real-life phenomena. Classically, CAs are uniform. However, non-uniformity has also been introduced in update pattern, lattice structure, neighborhood dependency and local rule. In this survey, we tour to the various types of CAs introduced till date, the different characterization tools, the global behaviors of CAs, like universality, reversibility, dynamics etc. Special attention is given to non-uniformity in CAs and specially, the non-uniform elementary CAs, which have been very useful in solving several real-life problems.

CCS Concepts: •**Theory of computation** → **Computability**; **Abstract machines**; *Formal languages and automata theory*; *Theory and algorithms for application domains*;

General Terms: Cellular Automata

Additional Key Words and Phrases: Non-uniform CA, Elementary CA, Global Behavior, Neighborhood, State, Rule, Characterization Tool

1. INTRODUCTION

From the end of the first half of 20th century, a new approach has started to come in scientific studies; which after questioning so called Cartesian analytical approach, says that interconnections among the elements of a system, be it physical, biological, artificial or any other, greatly effect the behavior of the system. In fact, according to this approach, knowing the parts of a system, one can not properly understand the system as a whole. In physics, David Bohm is one of the advocates of this approach [Bohm 1980]. This approach is adopted in psychology by Jacob Moreno [see as example [Moreno 1932]], which later gave birth of a new branch of science, named Network Science. During this time, however, a number of models, respecting this approach, have started to be proposed [see, as an example, [McCulloch and Pitts 1943]]. Cellular Automata (CAs) are one of the most important developments in this direction.

The journey of CAs was initiated by John von Neumann for the modeling of biological *self-reproduction* [Neumann 1966]. A cellular automaton (CA) is a discrete dynamic framework comprising of a orderly network of cells, where each cell is a finite state automaton. The next state of the cells are decided by the states of their neighboring cells following a local update rule. John von Neumann's CA is an infinite 2-dimensional square grid, where each square box, called cell, can be in any of the possible 29 states. The next of each cell depends

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on the state of itself and its four neighbors. This CA can not only model biological self-reproduction, but also is computationally universal [Thatcher 1964]. The beauty of a CA is, simple local interaction and computation of cells results in a huge complex behavior when the cells act together.

Since their inception, CAs have captured the attention of a good number of researchers of diverse fields. They have been flourished in different directions – as universal constructors [Thatcher 1964; Banks 1970], models of physical systems [Chopard and Droz 1998], parallel computing machine [Toffoli and Margolus 1987], etc. In recent years, the CAs are highly utilized as technology. As a consequence, a number of variations of the CAs have been developed. In this survey, we will conduct a tour to a few aspects of CAs. Our purpose is not to teach one in depth about any specific aspect of CAs, rather to cover different directions of CAs research along with a good number of references. Interested readers may go to the appropriate references to explore their interests in depth.

We conduct this survey under six categories. First, we tour the various types of CAs, developed since their inception (Section 2). For example, von Neumann’s CA and Wolfram’s CA [Wolfram 2002] are of different types – first one is defined over 2-D grid having 29 states per cell, whereas latter is one-dimensional binary CA. Second, to study the behavior of CAs, few tools and parameters are developed. They are visited in Section 3. However, the most interesting property of CAs is probably their complex global behavior due to very simple local computations of the cells. We inspect some of the most explored global behavior in Section 4.

During last two decades, a new direction in CAs research has been opening up, which introduces non-uniformity in CA structure. Practically, CAs are uniform in all respect - uniformity in cell’s update (that is, synchronicity), uniformity in neighborhood dependency, and uniformity in local rule. Though the *uniform* CAs are very good in modeling physical systems, researchers have introduced *non-uniformity* in CA structure to model some physical systems in a better way, and most importantly, to solve some real life problems efficiently. We survey various aspects of non-uniformity in Section 5, and put our deeper attention on a special type of non-uniform CAs in Section 6.

Nowadays, CAs are not only modeling tools, but also technologies. Use of CAs in different application domains has established this fact in the last two decades. We explore some of such applications in Section 7.

2. TYPES OF CELLULAR AUTOMATA

Classically, CAs are dynamic frameworks which are congruent and discrete with respect to time and space. At each time step t , cells of a CA change their states depending on the states of their neighbors at time $t-1$, following the same local update rule, in a synchronous way. Formally, a CA can be defined as a tuple $(\mathcal{L}, \mathcal{S}, \mathcal{N}, f)$, in which

- $\mathcal{L} \subseteq \mathbb{Z}^D$ is the D -dimensional cellular space; each element of \mathcal{L} is called a *cell*.
- Set \mathcal{S} is finite, where the elements are called *states* that each cell assumes at any time t .
- $\mathcal{N} = (\vec{v}_1, \vec{v}_2, \dots, \vec{v}_N)$ is the neighborhood vector of N distinct elements of \mathcal{L} which associates one cell to its *neighbors*. A cell at location $\vec{v} \in \mathcal{L}$ have N neighboring cells at locations $(\vec{v} + \vec{v}_i) \in \mathcal{L}$, $1 \leq i \leq N$.
- $f : \mathcal{S}^N \rightarrow \mathcal{S}$ is called the local rule of the automaton. The next state of a cell is given by $f(a_1, a_2, \dots, a_N)$, where a_1, a_2, \dots, a_N are the states of its N neighbors.

The mapping $c : \mathcal{L} \rightarrow \mathcal{S}$ implies the configuration of the automaton indicating the states of every cell at time t . \mathcal{C} represents $\mathcal{S}^{\mathcal{L}}$, that is the set of all configurations. After one time stamp, the next configuration of c becomes $c' = G(c)$, where $c'(\vec{v}) = f(c(\vec{v} + \vec{v}_1), c(\vec{v} + \vec{v}_2), \dots, c(\vec{v} + \vec{v}_N))$, for all $\vec{v} \in \mathcal{L}$; $G : \mathcal{C} \rightarrow \mathcal{C}$ is called the *global transition function* of the CA.

However, different types of CAs can be identified by varying the properties of $\mathcal{L}, \mathcal{S}, \mathcal{N}$ and f . We reorganize the parameters of CAs under the following headings –

- (1) the dimension and neighborhood of a cell,
- (2) the states of the cell,
- (3) the lattice size and boundary condition and
- (4) the local transition rule.

Next, we discuss the types of CAs under the above headings.

2.1. The dimension and neighborhood of a cell

The neighborhood of CA cells is strongly correlated with the dimension of the CAs. The original CA, proposed by von Neumann, is of 2-dimension which uses 5 (orthogonal ones and itself) neighborhood dependency and 29 states per cell. Later, the research on this field was conducted in mainly two directions.

– In the first direction, new neighborhoods concepts have been proposed. In [Moore 1962], a 9-cell or unit cube neighborhood, termed as Moore neighborhood, was proposed (see Fig. 1d). This structure has been utilized to design the famous *game of life*, introduced by John Conway and popularized by Martin Gardner [Gardner 1971]. In 2-dimensional CAs, some other variations, such as Margolus neighborhood [Toffoli and Margolus 1987], are also reported. In [Ghosh et al. 2012], a specialized class of CA, named Restricted 5-Neighborhood CA is reported to deal with identification of Protein Structure. In [Morita et al. 1999; Siap et al. 2011], hexagonal CAs are defined over 2-D grid where each cell is of the form of a hexagon. CAs are defined in hyperbolic plane in [Margenstern and Morita 1999; Margenstern and Morita 2001]. CAs are also defined over networks, called Automata Network (see Section 5.2), where the neighborhood dependencies of different cells may be different [Tomassini et al. 2005; Darabos et al. 2007].

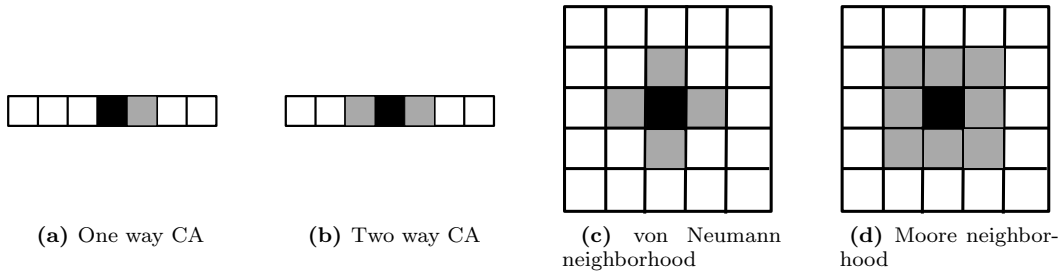


Fig. 1: 1a and 1b are 1-dimensional CAs, and 1c and 1d are 2-dimensional CAs.

Sometimes, neighborhood of a CA is also represented by *radius*. By *radius*, we mean the number of consecutive cells in a direction on which a cell depends on. For example, for 1-dimensional CA, neighborhood \mathcal{N} can be represented as $\mathcal{N} = \{-r, -(r-1), \dots, -1, 0, +1, \dots, +(r-1), +r\}$, where r is the radius or range of the CA. Classically size of the neighborhood $N = 2r + 1$ and radius is same in every direction. However, in some works this restriction is removed, such as [Jump and Kirtane 1974; Boccarra and Fuks 1998]. One of such CAs, called *one-way CA*, has been proposed in which communication is allowed only in one direction, that is, in a 1-dimensional array, the next state of each cell depends on itself and either of its left neighbor(s) or right neighbor(s) (see Fig. 1a) [Dyer 1980].

– In the second direction, research is continued either to study the CAs defined over 2 or higher dimension, or to explore the simplified 1-dimensional CAs. In [Packard and Wolfram 1985; Durand 1993; Terrier 2004; de Oliveira and Siqueira 2006; Uğuz et al. 2014], some fundamental properties and parameters are explored for 2-dimensional CAs. For example, in [Durand 1993], it is proved that, the decision algorithm, whether a 2-D CA is injective or not, when restricted to finite configurations not greater than its length, is a co-NP-complete problem. Nevertheless, in [Gandin and Rappaz 1997; Sarkar and Barua 1998; Miller and Fredkin 2005; Mo et al. 2014], higher than 2-dimensional CAs are proposed. For instance, in [Miller and Fredkin 2005] two-state three-dimensional reversible CA (RCA) is described and is shown to accomplish universal computation and construction.

Nevertheless, a large number of researches are dedicated to explore one-dimensional CAs; see for example [Amoroso and Patt 1972; Pries et al. 1986; Sutner 1991; Sikdar et al. 2002; Das and Sikdar 2009]. Wolfram in 1980s, introduced very simple CAs, called *elementary* CAs or ECAs, which are one-dimensional, two-state, and having three-neighborhood dependency [Wolfram 1994; Wolfram 2002]. However, 1-D CAs sometime behave differently than 2 or higher dimensional CAs. For example, reversibility, an well addressed problem of 1-D CA, is undecidable for 2 or higher dimensional CA [Kari 1990].

2.2. The states of the cell

A cell can be in any of the states of a finite state set \mathcal{S} at any point of time. Initially, many researchers concentrated to simplify von Neumann's CA with regard to the number of states. For instance, state count of CAs was reduced in [Lee 1964] and further reduced to eight in [Codd 1968]. In [Thatcher 1964], construction and computational universality as well as self-reproducing ability of von Neumann's cellular space is shown, whereas [Arbib 1966] depicted a simple self-reproducing CA capable of universality with only 2 states. [Banks 1971] has proved the universal computability of 2-state CA. However, all these constructions were for 2-dimensional infinite CAs, and used 5-neighborhood dependency (see Fig. 1c). The most simplified CAs, elementary CAs (ECAs) are proposed by Wolfram on 1-dimension having two states and 3-neighborhood interconnection.

Sometimes a state $q \in \mathcal{S}$ is called *quiescent* state. Next state of a cell is said to be in the quiescent state, if each neighbor of it is in the quiescent state, that is, if $f(q, q, \dots, q) = q$. This state represents the stability of the system. If a configuration of a CA has only a finite number of non-quiescent cells, then it is called a finite configuration, otherwise it is infinite.

However, in [Smith 1971a], Smith has shown that neighborhood size and state-set cardinality of a CA are interrelated. A CA with higher neighborhood size can always be emulated by another CA that has lesser neighborhood size but higher number of states per cell, and the contrariwise.

Traditionally, every cell of a CA follows the same state set. However, a CA may have different state sets, called polygeneous CA [Sarkar 2000], which received attention in the work of Holland [Burks 1970]. In linear/additive CA, some works are recorded considering the state to be elements of a finite field [Martin et al. 1984; Chaudhuri et al. 1997], and an extension field [Sikdar et al. 2000; Sikdar et al. 2002], and some considering the state set to be \mathbb{Z}_m (the integers modulo m) [Itô et al. 1983].

2.3. The boundary condition

Likewise neighborhood condition and dimension of CA, which are strongly correlated, lattice size and boundary conditions are also interdependent. Although ideally, a CA contains infinite cells of grid, but for the sake of simplification, it is assumed to be a collection of the finite grid. Hence, a boundary is imposed to the finite CAs. Generally two boundary conditions are assumed - open boundary condition and periodic boundary condition.

–In the open (fixed) boundary CAs, the extreme cells (leftmost and rightmost cells in case of 1-D CAs) are assigned some fixed states. Among the open boundary conditions,

the most popular is null boundary, where both the missing neighbors of the leftmost and rightmost terminal cells are always in state 0 [Chaudhuri et al. 1997].

—In periodic boundary CAs, the boundary cells are neighbors of some other boundary cells. For instance, for 1-D CAs, the rightmost and leftmost cells are neighbors of each other. In [Sarkar and Barua 1998; Jin and Wu 2012; Uğuz et al. 2013] etc., higher dimensional CAs are explored under periodic boundary condition.

Some other variations of boundary condition also exist, such as adiabatic boundary, reflexive boundary and intermediate boundary. Adiabatic boundary is where the missing neighbors assume states of the boundary cell, whereas in reflexive boundary condition, the left and right neighbors have the same value as the boundary cells [Uğuz et al. 2014]. However, for intermediate boundary condition, the cell value present at the previous to previous (next to next) cell of the left- (right-) most cell acts as the left (right) boundary [Chaudhuri et al. 1997]. As another special case of open boundary condition, stochastic boundary condition is proposed, where the boundary cells assume some states stochastically.

2.4. Local rule

A cell of a CA changes its state following a next state function $f : \mathcal{S}^N \rightarrow \mathcal{S}$ where \mathcal{S} is the set of states and N is the size of the neighborhood. The f , which is generally known as the local rule of the CA, can be conveyed in different ways. As an instance, in Conway's Game of Life, the local rule is $f : \{0, 1\}^9 \rightarrow \{0, 1\}$, where state 0 represents a dead cell and state 1 represents an alive cell. It is stated as following in [Li et al. 2010]:

- “Birth: a cell that is dead at time t , will be alive at time $t + 1$, if exactly 3 of its eight neighbors were alive at time t .
- Death: a cell can die by:
 - Overcrowding: if a cell is alive at time t and 4 or more of its neighbors are also alive at time t , the cell will be dead at time $t + 1$.
 - Exposure: If a live cell at time t has only 1 live neighbor or no live neighbors, it will be dead at time $t + 1$.
- Survival: a cell survives from time t to time $t + 1$ if and only if 2 or 3 of its neighbors are alive at time t .”

However, the local rule f can also be represented in a tabular form. In this form, the table contains entries for the next state values corresponding to each of the possible neighborhood combinations according to the local rule. For example, Table I represents seven elementary CA rules, where the entries in columns 2 to 9 in the 1st row depicts all the possible 3 neighborhood combinations. Wolfram in [Wolfram 1983] has introduced a naming scheme for elementary CAs, in which the 8-bit sequence from $f(111)$ to $f(000)$ is named as the CA rule and represented by its decimal equivalent. In Table I, the last column represents the decimal equivalent of the rules. Obviously, the tabular form is good if size of neighborhood (N) and the state set (\mathcal{S}) are very small.

Sometimes, the function representing the local rule of a CA is linear, which can be expressed as

$$f(a_1, a_2, \dots, a_N) = c_1 a_1 + c_2 a_2 + \dots + c_N a_N,$$

where the state set \mathcal{S} is a commutative finite ring with identity” [Kari 2005b] and $c_i \in \mathcal{S}$ is a constant. Such CAs are called linear or additive. There are seven linear/additive ECAs for rules 60, 90, 102, 150, 170, 204 and 240 (excluding rule 0, which is also trivially linear/additive).

Classically, the rules are uniform, synchronous and deterministic – that is, all cells are updated together with the same local rule. A new class of CAs have been proposed where cells can follow different rules. In that case, the local rules are represented by a rule vector \mathcal{R} , where \mathcal{R} contains entries of local rule for each cell. Such CAs are called non-uniform or

Table I: Rules of elementary CAs. Here, PS and NS represent present state and next state respectively

PS	111	110	101	100	011	010	001	000	
(RMT)	(7)	(6)	(5)	(4)	(3)	(2)	(1)	(0)	Rule
	0	1	0	1	1	0	1	0	90
	1	0	0	1	0	1	1	0	150
	0	0	0	1	1	1	1	0	30
NS	0	0	0	0	0	1	0	1	5
	0	1	0	0	1	0	0	1	73
	1	1	0	0	1	0	0	0	200
	0	1	0	1	0	0	0	0	80

hybrid CAs [Pries et al. 1986; Hortensius et al. 1989; Chaudhuri et al. 1997]. Another type of CA, called a programmable CA (with respect to VLSI design), exists, where a cell can chose a distinct local rule at every time instant [Nandi et al. 1994]. Nevertheless, a special kind of CA is also introduced where a CA updates itself in asynchronous way [Fatès 2014]. Section 5 and Section 6 are dedicated to discuss these non-classical CAs.

3. CHARACTERIZATION TOOLS OF CELLULAR AUTOMATA

In *A New Kind of Science*, Stephen Wolfram argued that, to find out how a particular CA will behave, one has to observe what is happening just by running the CA. Predicting behavior of a system by means of (mathematical) analysis and without running it is only possible, according to Wolfram, for special systems with simple behavior (page 6 of [Wolfram 2002]). In spite of this observation of some CAs researchers, a few characterization tools and parameters have been proposed in different time to analyze and predict the behavior of some CAs. Needless to say, all kind of dynamic behaviors of a CA may not be analyzed by a tool, but tools are used to discover some specific properties of a CA.

In this section, we survey the characterization tools and parameters, developed till date to analyze the CAs. Tools are mainly developed for one-dimensional CAs, and for two or higher dimensional CAs, “run and watch” is the primary technique to study the behavior. Though, few parameters are proposed which can be used to guess the behavior of two or higher dimensional CAs.

3.1. de Bruijn graph

In the different development phases of CAs, graph theory has played an important role. One of the roles is describing the evolution of an automaton, and another is relating local properties to global properties. As an automaton has states which are mapped to another states using overlapping neighborhood sequence, it is very obvious to treat it by shift register. So, de Bruijn graph is considered as an alternative characterization tool.

An m -dimensional de Bruijn graph of k symbols is a directed and edge-labeled graph representing overlaps between sequences of symbols. It has k^m vertices, consisting of all possible length- m sequences of the given symbols. In general, the de Bruijn graph $B(k, m) = (V, E)$, where k represents the number of symbols and m is the dimension, has k^m vertices (V) and k^{m+1} edges (E). The graph is balanced in the sense that each vertex has both in-degree and out-degree k [de Bruijn 1946].

A CA can be expressed as a de Bruijn graph $B = (\mathcal{S}, m, f)$ of dimension $m = N - 1$, where N depicts the size of the neighborhood, over the state set \mathcal{S} in which each edge $e \in \mathcal{S}^{m+1}$ is labeled by $f(e)$ - f is the CA rule. For example, for a 1-dimensional 3-neighborhood 2-state CA ($\mathcal{S} = \{0, 1\}$), the graph will have $2^2 = 4$ vertices and $2^3 = 8$ edges. Each edge is labeled with xyz/u where xyz represents a sequence of 3 symbols from \mathcal{S} that comes

from the overlap of labels of the two nodes of that directed edge and $u = f(x, y, z)$. The de Bruijn graph of Fig. 2 represents one such CA. This graph shows that if the left, self and right neighbors of a cell are all 0s, then next state of the cell (that is, $f(0, 0, 0)$) is 0, if the neighbors are 0, 0 and 1 respectively, the next state is 1, and so on.

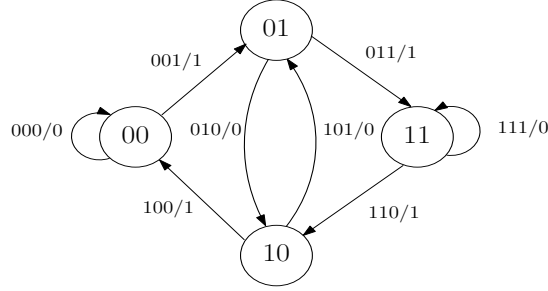


Fig. 2: The de Bruijn Graph of CA with rule 90 (2^{nd} row of Table I)

Over the years, this graph has been used by various researchers to understand global behavior, like surjectivity and reversibility, number conservation and equicontinuity etc. of CAs [Sutner 1991; Soto 2008; Voorhees 2008]. In [Martinez et al. 2008], ECA 110 has been explored to determine a glider-based regular expressions. In [Mora et al. 2008], cyclic properties and inverse of a CA are characterized using pair diagram of de Bruijn graph.

The de Bruijn graphs are traditionally used to represent and study classical CAs, though few works are recently reported where de Bruijn graphs are utilized to represent non-uniform or hybrid CAs [Provillard et al. 2011]. However, another tool, named *Reachability tree*, has also been recently developed, which can represent as well as discover the properties of a non-uniform CA in a better way.

3.2. Matrix algebra

Matrix algebra is a well-known characterization tool proposed for finite CAs [Das et al. 1990; Das et al. 1992; Chaudhuri et al. 1997]. This tool, however, works for only a particular type of CAs, called *linear/additive* CAs. The speciality of matrix algebra is, it can characterize non-uniform CAs. In fact, matrix algebra was first used in [Das et al. 1990] to characterize 1-D binary non-uniform CAs.

The linear rules of binary CAs can be expressed by XOR logic; see, for example, Table III, which is reproduced directly from [Chaudhuri et al. 1997]. Such a CA of size n is characterized by an $n \times n$ characteristics matrix operating on $GF(2)$, where the i^{th} row of the matrix represents the dependency of the i^{th} cell to its neighbors. The characteristics matrix (T), in this case, is formed as [Chaudhuri et al. 1997]:

$$T[i, j] = \begin{cases} 1 & \text{if the next state of the } i^{th} \text{ cell depends on the present state of the } j^{th} \text{ cell} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

The non-uniform CAs use different rules for different cells, hence we need a *rule vector* to specify the rules against cells (see Section 6). Let us take a 4-cell non-uniform elementary CA with rule vector $\mathcal{R} = \langle 150, 150, 90, 150 \rangle$ under null boundary condition. Then, the

characteristics matrix of the CA is:

$$T = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Here, third cell's rule is 90, hence depends on left and right neighbors only (see Table III). So, the third row of the T is $[0 \ 1 \ 0 \ 1]$. Since boundary condition is null, left neighboring cell of first cell and right neighboring cell of last cell are missing. The next configuration of the CA is incurred by multiplying the matrix with the present configuration vector, that is $S(t+1) = T \cdot S(t)$, $S(t)$ is the configuration of the CA at time t . Note that, the matrix T can be used to efficiently characterize the state-transition behavior, reversibility properties etc. of a linear CA; for more detailed discussion please see [Chaudhuri et al. 1997].

In [Sikdar et al. 2000; Sikdar et al. 2001; Sikdar 2003], matrix algebra is used to characterize $GF(2^p)$ CA, designed over the extension of Galois field $GF(2^p)$, where the state set $\mathcal{S} = \{0, 1, 2, \dots, 2^p - 1\}$. Further, hierarchical CA over $GF(2^{p^{q^{\dots}}})$ is also designed [Sikdar et al. 2002]. Recently, reversibility is studied for the pentadiagonal rule matrix in \mathbb{Z}_2 with null boundary condition [Martín del Rey and Rodríguez Sánchez 2011] and linear CA of radius 2 with periodic boundary conditions over \mathbb{Z}_p , where $p \geq 2$ is a prime number [Cinkir et al. 2011]. For 2-dimensional linear CAs also, a list of works are reported using matrix algebra as the characterization tool [Chattopadhyay et al. 1999; Siap et al. 2011; Uğuz et al. 2013; Uğuz et al. 2014].

3.3. Reachability tree

Reachability tree is an efficient tool of characterizing one-dimensional finite CA, which represents the reachable configurations of the CA. A configuration is called non-reachable, if it has no predecessor, that is, can not be reached from any configuration of the CA. The strength of this tool is, it can characterize any non-linear, non-uniform CAs with ease [Das et al. 2004; Das and Sikdar 2006; Das et al. 2008; Das 2007]. Initially, this tool was introduced for only 2-state CAs. However, in [Bhattacharjee and Das 2016], it has been shown that, the tree can also characterize any d -state CA, $d \geq 2$.

For 2-state n -cell CA, the reachability tree is a binary tree having $n+1$ levels where each level i can contains maximum 2^i nodes. The node at level 0 is called root and nodes of level n are called leaves. Each of the nodes and edges are labeled with a set of *RMTs*.

To understand RMTs, please refer to de Bruijn graph of Fig. 2 of Section 3.1. Recall that, in this figure, each edge is labeled by xyz/u where xyz is all possible neighborhood combinations. Each of these edge labels xyz is named as *Rule Min Term* or *RMT* in the reachability tree. So, RMT is the combination of the neighborhoods x, y, z against the value $f(x, y, z)$, where $f : \mathcal{S}^3 \rightarrow \mathcal{S}$ is the local rule. Each RMT is associated to a number $r_d = x \times d^2 + y \times d + z$ and the value $f(x, y, z)$ is denoted by $f[r_d]$. Therefore, for a d -state 3-neighborhood CA, number of RMTs is d^3 . In the tabular representation of local rule in Table I, each of the columns 2 – 9 of 1st row is an RMT, represented by their decimal equivalents.

However, the RMTs are related to each other. For instance, the set of RMTs from the incoming edges of Fig. 2 are called *equivalent* RMTs, whereas, the set of outgoing RMTs are termed as *sibling* RMTs. RMT i and all of its equivalents form a set $Equi_i$ and similarly, the set of sibling RMTs form $Sibl_j$, $0 \leq i, j \leq d^2 - 1$. For a d state CA, there are d^2 sets of equivalent RMTs and d^2 sets of sibling RMTs. Table II shows the relationship among the RMTs of 2-state CAs.

A configuration of a CA can be depicted by a sequence of RMTs, called *RMT sequence*. For example, let 1101 be a configuration of a 4-cell CA. Then the RMT sequence corre-

Table II: Relationship between RMTs of 2-state CAs

Incoming			Outgoing		
#Set	Equivalent RMTs	Decimal Equivalent	#Set	Sibling RMTs	Decimal Equivalent
$Equi_0$	000, 100	0, 4	$Sibl_0$	000, 001	0, 1
$Equi_1$	001, 101	1, 5	$Sibl_1$	010, 011	2, 3
$Equi_2$	010, 110	2, 6	$Sibl_2$	100, 101	4, 5
$Equi_3$	011, 111	3, 7	$Sibl_3$	110, 111	6, 7

sponding this configuration is $\langle 3(011), 6(110), 5(101), 2(010) \rangle$, if the CA is on null-boundary and $\langle 7(111), 6(110), 5(101), 3(011) \rangle$, if the CA is on periodic boundary. To get a RMT sequence, an imaginary window of length 3 is considered, which slides over the configuration, one cell right at a time. For the i^{th} RMT in the sequence, this window contains the states of $(i-1)^{th}$ cell, i^{th} cell and $(i+1)^{th}$ cell. The decimal value corresponding to this 3-cell window is the i^{th} RMT in the sequence. Note that, in case of null boundary, the terminal cells are taken as 0, and in periodic boundary condition, the grid is considered are circular.

Although, the structure of nodes and edges in reachability tree is different for different boundary conditions, but, in all the cases, the content of the root node is predefined. For example, in null-boundary, the root node of the tree of an ECA contains RMTs 0,1,2 and 3, and in periodic boundary condition, it contains all the sibling RMTs. From the root node, the edges are constructed following the CA rule; the RMTs for which the next state value is 0 in the rule, form 0-edge and the RMTs with next state value 1 form 1-edge. However, in case of non-uniform n -cell CAs with a rule vector $\mathcal{R} = \langle \mathcal{R}_0, \mathcal{R}_1, \dots, \mathcal{R}_{n-1} \rangle$, the edges of level i are contrived with the local rule \mathcal{R}_i . Formally, for null-boundary condition, the reachability tree is defined as the following [Naskar et al. 2014] :

Reachability tree for an n -cell CA under null boundary condition is a rooted and edge-labeled binary tree with $(n+1)$ levels, where $E_{i,2j} = (N_{i,j}, N_{i+1,2j}, l_{i,2j})$ and $E_{i,2j+1} = (N_{i,j}, N_{i+1,2j+1}, l_{i,2j+1})$ are the edges between nodes $N_{i,j}$ and $N_{i+1,2j}$ with label $l_{i,2j}$, and between nodes $N_{i,j}$ and $N_{i+1,2j+1}$ with label $l_{i,2j+1} \subseteq N_{i,j}$, resp. ($0 \leq i \leq n-1$, $0 \leq j \leq 2^i - 1$). The following relations are maintained in the tree:

- (1) $l_{i,2j} \cup l_{i,2j+1} = N_{i,j}$
- (2) $\forall r_d \in l_{i,2j}$ (resp. $\forall r_d \in l_{i,2j+1}$), $\mathcal{R}_i[r_d] = 0$ (resp. $\mathcal{R}_i[r_d] = 1$) and RMTs $2r_d \pmod{8}$ and $2r_d + 1 \pmod{8}$ of \mathcal{R}_{i+1} are in $N_{i+1,2j}$ (resp. $N_{i+1,2j+1}$)
- (3) $\bigcup_{0 \leq j \leq 2^i - 1} N_{i,j}$ is the set of all *valid* RMTs of \mathcal{R}_i , ($0 \leq i \leq n-1$)

Fig. 3 shows an example of reachability tree of CA with rule vector $\mathcal{R} = \langle 5, 73, 200, 80 \rangle$ under null boundary condition. In this figure, edges of level 0 are constructed by rule 5, level 1 with rule 73, level 2 with rule 200 and the edges of level 4 are constructed with rule 80. Note that, if an edge contains an RMT $r_d \in Equi_i$, then in the corresponding node all the RMTs of $Sibl_i$ are added, that is the nodes are constructed with the sibling RMTs generated from the RMTs of the corresponding edges.

The definition of reachability tree under periodic boundary condition are depicted in [Das and Sikdar 2009] for binary CAs and in [Bhattacharjee and Das 2016] for d -state CAs, $d \geq 2$. In general, reachability tree of an n -cell d -state CA is a rooted and edge-labeled d -ary tree with $(n+1)$ levels, where a sequence of edges from the root to leaf represents a reachable configuration and the corresponding RMT sequence depicts the predecessor configuration. If a node/edge is missing in the tree, then the edge-sequence represents a non-reachable configuration. Therefore, the reachability tree portrays all the reachable configurations of an n -cell CA. For example, in Fig. 3, number of reachable configurations is 8 out of $2^4 = 16$ and one such reachable configuration is the edge sequence $\langle E_{0,0}, E_{1,0}, E_{2,1}, E_{3,3} \rangle$ representing 0011. There are 2 RMT sequences $\langle 3, 7, 6, 4 \rangle$ and $\langle 3, 7, 7, 6 \rangle$, representing 1110 and 1111 as

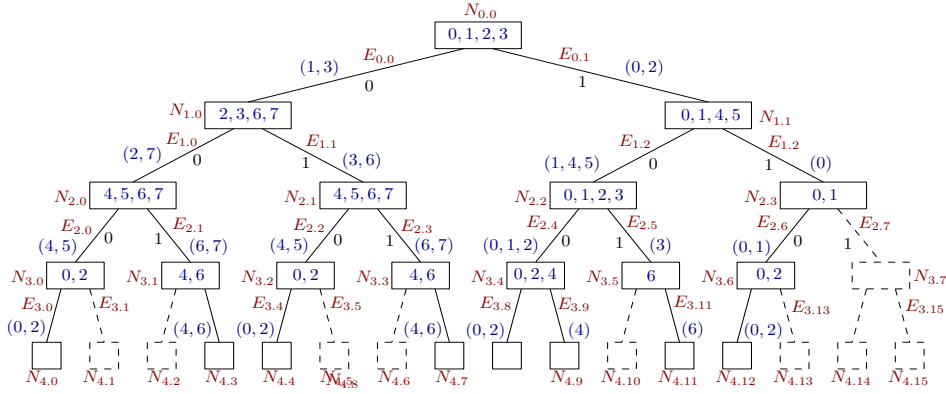


Fig. 3: Reachability Tree for null boundary 4-cell binary CA with rule vector $\mathcal{R} = \langle 5, 73, 200, 80 \rangle$

the predecessor configurations, corresponding to this present configuration. Moreover, the edge sequence $\langle E_{0,0}, E_{1,0}, E_{2,0}, E_{3,1} \rangle$ represents a non-reachable configuration 0001. So, the CA is irreversible.

Reachability tree checks reversibility of any 3-neighborhood 2-state CA in linear time [Das 2007; Das and Sikdar 2009]. This characterization tool can also identify and count point state attractors with or without *identification bit* (Pseudo-Exhaustive bit) [Das et al. 2009a; Naskar et al. 2012b] etc. in time $\mathcal{O}(n)$.

However, it is observed that only reachability tree is not sufficient to address some problems. As a result, *links* are developed among the edges of the tree. These links help to discover state transitions in the tree [Naskar et al. 2012a]. A special type of link, called cross link, is also distinguished, which is related to multi-state attractor. The detail characterization of cross links helps to discern whether a given CA has only point state attractors in its configuration space [Naskar et al. 2014].

3.4. Z-Parameter, λ -Parameter, Θ -Parameter

There are some parameters that can be used to characterize some aspects of one-dimensional CAs, such as the λ parameter, the Z parameter, and the obstruction (Θ) parameter. The λ , Z , and Θ parameters are proposed respectively by Langton [Li et al. 1990; Langton 1990], Wuensche and Lesser [Wuensche 1994; Wuensche 1998] and Voorhees [Voorhees 1997]. For a d -state CA with radius r , if m out of the total d^{2r+1} neighborhood configurations map to a non-zero state, then λ is defined as: $\lambda = \frac{m}{d^{2r+1}}$, that is, the percentage of all the entries in a rule table which maps to non-zero states. This parameter can be compared with temperature in statistical physics, or the degree of non-linearity in dynamical systems, although these are not equivalent [Li et al. 1990].

To track behavior of binary CA, Z parameter is proposed as an alternate [Wuensche 1998]. It takes into account the allocation of rule table values to the sub-categories of related neighborhoods and predicts the convergence of global behavior, extremes of local behavior between order and chaos, surjectivity etc. From a given partial pre-image of a CA state, values of the successive cells can be deduced using this parameter. Two probabilities – Z_{left} and Z_{right} of the next unknown cell being deterministic are calculated from the rule-table. The Z parameter is the greater of these values and varies between 0 & 1. A derivation of the Z parameter in terms of rule table entries is also given in [Wuensche 1998]. High value of Z implies that number of pre-images of an arbitrary CA state is relatively small.

In [Voorhees 1997], the Θ parameter is defined and shown to characterize the degree of non-additivity of a binary CA rule. It is shown that the λ parameter and Θ parameter are

equal respectively to the area and volume under certain graphs. These parameters prove their utilization in classification of one-dimensional binary CAs.

3.5. Space-time Diagram and Transition Diagram

Although neither of space-time diagram and transition diagram (or state-transition diagram) is a characterization tool, but these two diagrams have been used to observe and predict the behavior and dynamics of a CA evolution. Both the diagrams are for finite CAs.

Space-time diagram is the graphical representation of the configurations of a CA at each time t . Here, the configuration lies on x -axis and y -axis represents time. Each of the CA states are generally depicted by some color (see Fig. 4a). So, the evolution of the CA can be visible from the pattern generated in the state-space diagram. This has been used to study the nature of CA in a set of papers, see for example [Wolfram 2002; Dennunzio et al. 2014]. Some packages are available in public domain which can be used to observe space-time diagrams of CAs - *Fiatlux* (<http://fiatlux.loria.fr/>) is one of them.

Transition diagram or State transition diagram [Chaudhuri et al. 1997] is defined as a “directed graph whose vertices are the configurations of the CA and whose edges represent one step evolution of the CA” [Sarkar 2000] (see Fig. 4b). This diagram is also called the basin-of-attraction field [Wuensche 1998]. It can be noted that, the dynamic behavior of any CA can also be visualized and studied in terms of its transition diagram. *Discrete Dynamics Lab*, an online laboratory (<http://www.ddlab.com/>), is a good place to get transition diagrams of CAs.

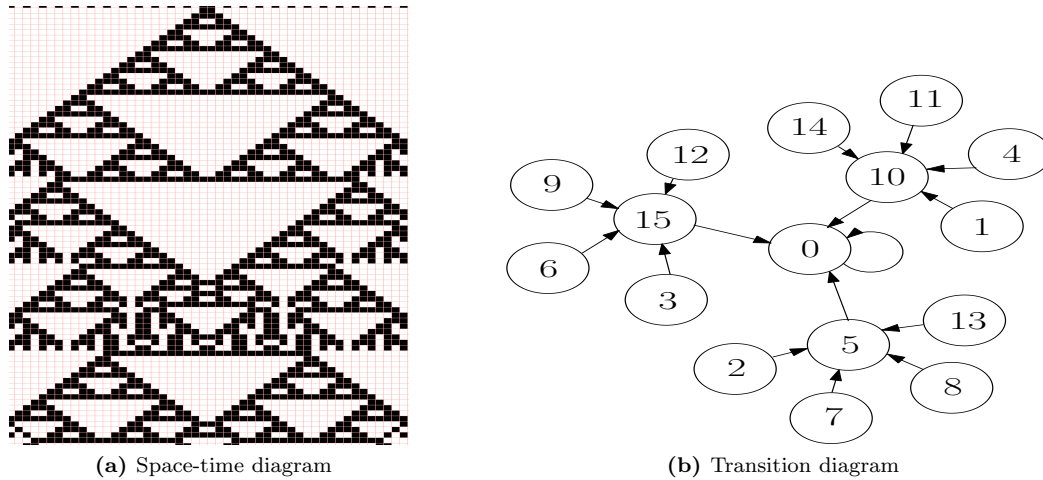


Fig. 4: Space-time diagram and transition diagram of ECA 90 (2^{nd} row of Table I) under periodic boundary condition. For (a), cell length $n = 51$, and black implies state 1 and white implies state 0. For (b), $n = 4$, and the configurations are in their decimal form.

4. GLOBAL BEHAVIOR OF CELLULAR AUTOMATA

The most exciting aspect of CAs is their complex global behavior, which are resulted from simple local interaction and computation. The CAs have many elemental properties of the real world, such as reversibility and conservation laws, chaotic behavior etc. These properties motivate researchers to simulate physical and biological systems like lattice gases [Frisch et al. 1986], Ising spin models [Grinstein et al. 1985], traffic systems [Kerner 2004; Matsukidaira and Nishinari 2003] etc., using CAs. Apart from the ability of modeling physical world [Vichniac 1984; Chopard and Droz 1998], the CAs are capable of performing several

computational tasks. In this section, we survey some of such global behaviors of CA such as universality, reversibility, conservation laws, computability etc.

4.1. Universality

The first feature of CAs that has attracted the researchers is the capability of CAs in performing universal computation. Here, the main concept is “to perform a step-by-step simulation of a single tape Turing Machine (TM)” [Sarkar 2000] by a CA [Thatcher 1964; Arbib 1966; Banks 1970]. However, the idea of simulating TM does not use the parallelism property of CA. In [Smith 1971b], the existence of computation-universal cellular spaces with small neighbor-state product is proved. Later works have shown the procedure of simulating a TM by reversible CA [Morita and Harao 1989; Dubacq 1995]. In [Culik et al. 1990], a notable result is shown which states, “there exists a universal CA A_U with 14 states, which can simulate step by step any CA whose initial configuration and local rule are encoded as an initial configuration of A_U ” [Sarkar 2000]. Computation universality of one-way CAs and totalistic CAs are also reported [Albert and Culik II 1987]. Computational universality is also shown following very simple rules, such as Game of Life [Gardner 1971], rule 110 (elementary CA) [Wolfram 1984; Cook 2004] and the billiard ball computer (block CAs) [Durand-Lose 1998]. The existence of computation-universal one-dimensional CA with 7-states per cell for radius $r = 1$ and 4-states per cell for $r = 2$ is proved in [Lindgren and Nordahl 1990]. However, if a CA can simulate all CAs of the same dimension, it is called intrinsically universal [Kari 2005b]. The smallest intrinsically universal CA in one-dimension is reported in [Ollinger 2003]. For two dimension, nonetheless, the same is a 2-state and 5-neighborhood CA [Banks 1970]. Some other notable related works are [Martin 1994; Ollinger 2002].

4.2. Invertibility and reversibility

For long, the questions of invertibility have been the major focus of research in CAs. If the global transition function G of a CA is one-to-one, the CA is termed as an *injective* CA. However, it is called *surjective* if the function is onto and bijective if G is both onto and one-to-one. The study of reversibility of CA was started with Hedlund [Hedlund 1969] and Richardson [Richardson 1972] - they independently proved that all one-to-one or injective CAs are reversible, which means, if each configuration has exactly one successor and one predecessor, the CA is reversible. The *Curtis - Hedlund - Lyndon* Theorem [Hedlund 1969] shows that a CA is called *bijective* or *reversible* if its global transition function is *bijective* and the inverse function is also a CA. In their seminal paper [Amoroso and Patt 1972], Amoroso and Patt have shown an effective way to decide reversibility of 1-dimensional infinite CA, on the basis of the local rule. In [Di Gregorio and Trautteur 1975], a decision algorithm is reported for CAs with finite configurations. An elegant scheme based on de Bruijn graph for deciding the reversibility of a one dimensional CA is presented in [Sutner 1991]. It was later shown that, designing an efficient algorithm that tests reversibility of an arbitrary CA, defined over two or more dimensional lattice, is undecidable [Kari 1990]. An interesting result is reported in [Toffoli 1977], which says, a $(D + 1)$ -dimensional reversible CA can simulate any D -dimensional CA. Later [Morita and Harao 1989] showed that a 1-dimensional reversible CA can simulate reversible Turing machines as well as any 1-D CA with finite configurations [Morita 1995]. These results indicate to “the existence of universal one-dimensional reversible CA, since reversible Turing machines can be computationally universal” [Kari 2005a]. Some other notable works on reversible CAs are [Bennett 1973; Maruoka and Kimura 1979; Culik 1987; Durand 1993; Kari 1994; Moraal 2000; Mora et al. 2006; Soto 2008; Morita 2008].

While studying the reversibility (i.e. injectivity) of infinite and finite CAs, one can identify (at least) the following four cases.

- (1) An infinite CA whose global function is injective on the set of “all infinite configurations”.
- (2) An infinite CA whose global function is injective on the set of “all *periodic* infinite configurations”. In one-dimension, if there exists $p \in \mathbb{N}$ such that $v_{i+p} = v_i$ for all $i \in \mathbb{Z}$, the configuration v is periodic, or more precisely, spatially periodic.
- (3) A finite CA whose global function is injective on the set of “all finite configurations of length n ” for all $n \in \mathbb{N}$.
- (4) A finite CA whose global function is injective on the set of “all finite configurations of length n ” for a fixed n .

However, the periodic configurations can be viewed as configurations of a (finite) CA that uses periodic boundary condition [Kari 2005b]. Therefore, case 2 and case 3 of the above are equivalent. Further, case 1 and case 2 are equivalent in one-dimensional CAs [Sato and Honda 1977]. Hence, in one-dimension, cases 1, 2 and 3 are equivalent, and the case 4 is different from them. So, the algorithms of [Amoroso and Patt 1972] and [Sutner 1991], which are the decision procedures for case 1, can decide reversibility of the one-dimensional CAs of cases 1, 2 and 3. However, in [Bhattacharjee and Das 2016], an algorithm is reported which can decide the reversibility of case 4 CAs in one-dimension.

4.3. Garden-of-Eden

One of the earliest discovered results on CAs was the *Garden-of-Eden* theorems by [Moore 1962] and [Myhill 1963], respectively. *Injectivity* and *surjectivity* properties of CA are correlated by these theorems. A configuration is named as a Garden-of-Eden configuration, if it does not have a predecessor; that is, if it is a *non-reachable* configuration. In 1962, Moore showed that the existence of mutually erasable configurations in a two-dimensional CA is sufficient for the existence of Garden-of-Eden configurations [Moore 1962]. He also claimed that existence of a configuration with more than one predecessors ensures existence of another configuration without any predecessor. However, in [Myhill 1963], the reverse was proved. He showed that the extant of mutually indistinguishable configurations is both necessary and sufficient for the extant of Garden-of-Eden configurations.

In [Amoroso and Cooper 1970], the equivalence between the existence of mutually erasable configurations and mutually indistinguishable configurations were established. This implied that the converse of Moore’s result is true as well. It was also shown in the paper that, for finite configurations both of the above conditions remain sufficient, but neither is then necessary. For finite configurations, a CA is irreversible if and only if a Garden-of-Eden configuration exists and is surjective, if and only if it is bijective [Amoroso and Cooper 1970]. So, the existence of Garden-of Eden configurations violates the injectivity property. The relation between global function of infinite CA and its restriction to finite configurations was established in [Richardson 1972]. Some other important results are listed in [Maruoka and Kimura 1976; Sato and Honda 1977; Toffoli and Margolus 1990; Kari 2005b].

Garden-of-Eden theorems for CAs have further been extended to Cayley graphs of groups [Machi and Mignosi 1993; Ceccherini-Silberstein et al. 1999]. In [Capobianco 2007], the surjectivity and surjunctivity of CA in Besicovitch topology is recorded. Note that, “the Besicovitch pseudodistance measures the density of the set where two configurations differ. The quotient space obtained by identifying configurations at distance zero, endowed with the induced metric” [Capobianco 2007], implies the Besicovitch topology. In [Margenstern and Morita 1999; Margenstern and Morita 2001], CA is defined in hyperbolic plane. In [Margenstern 2009], it is depicted that, the injectivity and surjectivity properties, proved by Moore and Myhill, are no longer valid for CA in the hyperbolic plane.

4.4. Topology, dynamics and chaotic behavior of CAs

The global function G , corresponding to a CA may be observed by dynamics of topology and chaos theory [Kari 2005b]. At the same, it is also reported that a CA is called equicontinuous if all local functions are at equicontinuity points [Kurka 1997]. “ G is called sensitive to initial conditions, or simply sensitive, if there are no equicontinuity points” [Kari 2005b]. Kurka also proposed to classify CAs according to their degree of equicontinuity [Kurka 1997]. However, in [Durand et al. 2003b], it is proved that most of the classes are undecidable. Some more works regarding classification of CAs are [Gilman 1987; Sablik 2008; Lena and Margara 2008]. A dynamical system is chaotic if following conditions are satisfied:

- (1) transitivity,
- (2) dense temporally periodic points, and
- (3) sensitivity to initial conditions.

In case of cellular automata, sensitivity is implied by transitivity and according to [Kari 2005b], the second condition is an open problem. These properties are first proved in [Devaney 1986]. A list of works regarding these are reported in [Mitchell et al. 1993; Cattaneo et al. 1999; Cattaneo et al. 2000]. If the number of single connected components of a CA is only one, or its limit set contains just one configuration, then it is called a nilpotent. For two or more dimension, the nilpotency of CAs is undecidable [Culik et al. 1990]. The same result for 1-dimensional CAs was proved by Kari [Kari 1992]. For the two-dimensional space, the generalization of the parameters – sensitivity, neighborhood dominance and activity propagation, is reported in [de Oliveira and Siqueira 2006].

The study of symbolic dynamics, with shift function and bi-infinite words was started by Hedlund in [Hedlund 1969]. He proved that, a function is the global transition function of a CA, if and only if it is continuous and commutes with the shifts and also showed that, a CA is bijective or reversible, if and only if the global transition function of it, is a bijection. Some more works to study the dynamical properties of CA, using expansivity, subshift, homomorphism automorphisms and endomorphisms are reported in [Blanchard and Maass 1997; Ward 1994]. [Shereshevsky 1992] defined the left and right Lyapunov exponents for 1-dimensional CAs. [Finelli et al. 1998] generalized the theory of Lyapunov exponents for D -dimensional CAs and proved that all expansive CAs have positive Lyapunov exponents for almost all the phase space configurations.

4.5. Randomness

Stephen Wolfram in [Wolfram 1985] introduced CAs as an excellent source of pseudorandomness. Massive parallelism, simplicity and locality of interactions of CAs, offer many benefits over other techniques, specially in case of hardware implementation. So, it has become an area of extensive research in VLSI circuit testing [Hortensius et al. 1989; Chaudhuri et al. 1997; Das and Sikdar 2010], Monte-Carlo simulations [Saraniti and Goodnick 2000], Field Programmable Gate Arrays [Comer et al. 2012], cryptography [Wolfram 1986a; Das and Chowdhury 2013; Das and Chowdhury 2011] etc. However, most of the works on pseudorandomness have been divided in mainly two directions:

- Most of the research have been going on in the first direction, to generate pseudo-random *numbers* using CAs. Here, generally an integer X_i is generated between zero and some number m (word size of the computer), where the fraction $U_i = \frac{X_i}{m}$ is the real number, uniformly distributed between 0 and 1. This number can be generated in several ways. For example, in [Wolfram 1986b], the sequence is generated by ECA 30 from the single cell with initial state 1 among all cells, initiated with state 0. However, in [Hortensius et al. 1989], the values are taken from parallel cells by combining ECA 90 and 150, arranged in a definite order in the lattice. Some other related works are [Tomassini et al. 2000; Alonso-Sanz and Bull 2009]. In [Sipper and Tomassini 1996; Wang et al. 2008], some optimization

algorithms are applied on CAs, whereas in [Guan and Zhang 2003; Guan and Tan 2004] dynamic behavior is allowed in the cells to generate the pseudo-random numbers.

–In the second direction, pseudo-random *patterns* are generated using CAs. Here, pattern means the configuration of a CA of length n , where each cell can take any of the CA states. Note that, in a pattern, individual cell values have significance. For example, a sequence $\langle 0110 \rangle$ can be treated as a number 6, but in case of pattern, it is “0110”. Some notable works using non-uniform CAs are [Ganguly et al. 2002c; Das et al. 2003; Das et al. 2004; Das et al. 2004; Das et al. 2005] etc. Here finding of the minimum cell length is important. For example, in [Das et al. 2005], a 45-cell CA based pseudo-random pattern generator (PRPG) is designed which beats all existing PRPGs.

4.6. Conservation law

Apart from reversibility, there exist other conservation laws, which are equally important in physics. Various direction of conservation (invariants) laws in CAs are reported in [Fredkin and Toffoli 1982; Pivato 2002; Boccara and Fukś 2002]. Among them number conserving CAs (NCCAs) are the most studied and used concept. NCCAs are defined with respect to the spatially periodic configurations and finite configurations. Boccara and Fukś have shown the necessary and sufficient conditions of a 1-D CA to be NCCA, initially for 2 states per cell [Boccara and Fukś 1998] and then for any arbitrary number of states [Boccara and Fukś 2002]. In [Pivato 2002], a general treatment to conserved quantities in 1-D CA is reported. Fukś has shown that motion representations can be constructed by 1-dimensional binary NCCAs. He has also extended the work to probabilistic CAs [Fukś 2004]. NCCAs in higher dimensions are also studied [Durand et al. 2003a]. Further, universality and other dynamics of NCCAs are explored [Moreira 2003; Formenti and Grange 2003].

NCCAs have appeared in the contexts like, the models of highway traffic [Nagel and Schreckenberg 1992; Nagel 1996; Matsukidaira and Nishinari 2003; Kerner 2004; Hazari et al. 2012], and particle conservation [Kohyama 1989; Kohyama 1991] etc. In 2-dimension, the Margolus CA is a number conserving CA [Margolus 1984]. Apart from NCCAs, additive conserved quantities in CAs are introduced and investigated in [Hattori and Takesue 1991]. In [Takesue 1995], a necessary and sufficient condition for a given CA rule to profess a staggered invariant is studied. In [Morita and Imai 1998; Morita et al. 1999], the computational universality of partitioned number-conserving (and reversible) CA is shown by simulating a universal counter machine. However, these CAs are not exactly the same as NCCAs, because reducing a partitioned CA to a non-partitioned one is not number preserving. The concept of extending conserved quantities to that of monotone quantities was presented in [Kurka 2003], considering CAs with vanishing particles.

In [Das 2011], non-uniform NCCAs are characterized. This paper has reported $O(n)$ time algorithms for verification and synthesis of an n -cell non-uniform NCCA. Further, number conservation property of ECA under asynchronous update was studied in [Hazari and Das 2014].

4.7. Computational tasks

CAs have been explored to perform several computational tasks, namely density classification task [de Oliveira 2013], shortest path problem [Adamatzky 1996], French flag problem [Herman and Liu 1973], early bird problem [Legendi and Katona 1986], generation of circles and parabola [Delorme et al. 1999] and firing squad synchronization problems [de Oliveira 2013].

In the CA literature, two domains related to computation are mostly studied: density classification task and synchronization problems. The problem statement of density classification can be summarized as follows - any initial configuration having more 1s (0s) than 0s (1s) must converge to all 1s (0s) configuration. However, it is impossible to solve this problem with 100% efficiency [Land and Belew 1995]. Because of the impossibility of solv-

ing the standard density classification task, research efforts have shifted towards finding the best rule which can solve the problem *almost* perfectly for one-dimensional CAs [Maiti et al. 2006; Kari and Le Gloannec 2012], two-dimensional CAs [Morales et al. 2001; de Oliveira and Siqueira 2006] and stochastic CAs [Fatès 2013]. However, Fukś showed that the density classification task is solvable by running in sequence the trivial combination of elementary rules 184 and 232 [Fukś 1997].

The synchronization problems, like *firing squad*, *queen bee*, *firing mob* [Culik and Dube 1991] etc. are studied by CAs. The goal of firing squad synchronization problem is designing a CA, which initially has one active cell and evolves to a state, where every cell is simultaneously active [Noguchi 2004; Umeo et al. 2005]. The queen bee and leader election may be assumed as the inverse problem of firing squad problem. Here, initially states of all cells are identical and by choosing a proper rule, a cell comes to a special state [Smith 1976; Mazoyer et al. 1999; Beckers and Worsch 2001; Stratmann and Worsch 2002].

Another computation problem, named early bird problem was defined and investigated first by [Rosenstiehl et al. 1972]. Here, any cell in the quiescent state may be excited by the outside world. These excitations result in special bird states instead of the quiescent states. The task is to give a transition function such that, after a certain time the first excitation(s) can be distinguished from the later ones. This problem is well explored by a group of researchers [Vollmar 1977; Legendi and Katona 1981; Legendi and Katona 1986]. Some other computation problems, like French flag problem [Herman and Liu 1973], shortest path problem [Adamatzky 1996], generating discretized circles and parabola in real time [Delorme et al. 1999] etc. are also discussed in literature.

5. NON-UNIFORMITY IN CELLULAR AUTOMATA

Conventionally, all the variants of CAs possess basic three properties - *uniformity*, *synchronicity* and *locality*. The uniformity refers to that each of the CA cells are updated by the identical local rule. The synchronicity implies that all the cells are updated simultaneously; whereas locality refers to that the rules act locally and neighborhood dependencies of each cell is uniform. Note that, the cells perform computation locally, and the global behavior of CA is received due to this local computation only. However, synchronicity is a special type of uniformity, where all the cells are updated simultaneously and uniformly. In fact, uniformity is everywhere in CA, in local rule, cell update and in lattice structure. We can summarize this in the following way:

- Uniformity in update: all cells are updated simultaneously in each discrete time step.
- Uniformity in lattice structure and neighborhood dependency: lattice structure is uniform and each cell follows similar neighborhood dependency.
- Uniformity in local rule: each of the cells updates its state following the same rule.

Over the years, researchers have successfully been using classical CA as a modeling tool. However, it has become apparent that many phenomena, such as chemical reactions occurring in a living cell, are found in nature which are not uniform. These new modeling requirements led to a new variant of the CA. As a result, non-uniformity in CAs has been introduced. Following are the main three variants of non-uniformity in CAs, which we get after relaxing above mentioned constraints of uniformity.

- (1) Asynchronous cellular automata (ACAs): the cells are not updated at the same (discrete) time step and can be independently updated - breaks the uniform update constraint (discussed in Section 5.1).
- (2) Automata Network: the CA is on a network and the states of the node evolve with neighborhood defined by the network - breaks uniform neighborhood constraint (discussed in Section 5.2).

- (3) Hybrid or non-uniform cellular automata: cells can assume different local transition functions - breaks uniform local rule constraint (discussed in Section 5.3).

5.1. Asynchronous Cellular Automata (ACAs)

Like other synchronous systems, a CA also assumes a global clock which forces the cells to get updated simultaneously. This assumption of global clock is not very natural, and relaxed in ACAs. The concept of ACAs and their computational ability were first developed by Nakamura [Nakamura 1974], it was further studied in [Golze 1978; Nakamura 1981; Hemmerling 1982; Ingerson and Buvel 1984; Le Caër 1989]. ACAs were developed on two-dimensional grid in [Cori et al. 1993] to report the concurrent situations emerged in distributed systems.

The word ‘asynchronism’ means that the parts of the system do not share the same time. In asynchronous CAs, cells are independent and so, during the evolution of the system, the cells are updated independently. There are several interpretations on the way of applying asynchronism. By simplifying, it can be said that asynchronism is to break the perfect update scheme. The main asynchronous updating schemes found in literature are *fully asynchronous updating* and *α -asynchronous updating* schemes [Nakamura 1981; Le Caër 1989; Bouré et al. 2012; Dennunzio et al. 2012a]. [Dennunzio et al. 2013] developed an *m*-asynchronous CA and generalized the various updating methods used so far. A good survey on asynchronous update scheme can be found in [Fatès 2014].

In one of the pointing work, [Hemmerling 1982] has shown the computation equivalence of synchronous and asynchronous cellular space. He has also shown that, any d -state deterministic rule can be simulated by a $3d^2$ state asynchronous rule with same neighborhood dependency. From a different point of view, [Golze 1978] has shown that, a $(D+1)$ -dimensional asynchronous rule can simulate a D -dimensional synchronous rule. [Dennunzio et al. 2012a] showed how fully asynchronous CA can simulate universal turing machine.

ACAs are also used as models of concurrency and distributed systems [Cori et al. 1993]. The computing ability of ACAs of [Cori et al. 1993] has further been investigated in [Pighizzini 1994; Droste et al. 2000]. In [Golze and Priese 1982], ACAs are showed to be equivalent to asynchronous concurrent petri nets.

In [Blok and Bergersen 1999], the change that occurs in the Game of Life, when the sites get updated with a given probability, are identified. [Ruxton and Saravia 1998] have analyzed the sensitivity of ecological system modelled by simple stochastic cellular automata to spatio-temporal ordering. In [Tomassini and Venzi 2002; Capcarrere 2002], asynchronous rules are used to resolve the density classification problem and the global synchronization problem respectively. [Suzudo 2004] studied the usage of genetic algorithms for determining the mass-conservative (also called number-conserving) asynchronous models that would generate nontrivial patterns. The strange phenomenon of coalescence is described in [Fatès 2014]. Reversibility of ACAs is re-examined and reported in [Sarkar et al. 2012; Sethi et al. 2014]. In [Manzoni 2012], the dynamical properties of CAs, such as injectivity, surjectivity, permutivity, sensitivity, expansivity, transitivity, dense periodic orbits and equicontinuity are re-defined for the asynchronous CAs. Recently, Das and his colleagues [Sethi and Das 2015; Sethi et al. 2016] have designed an efficient two class classifier utilizing ACA.

5.2. Automata Network

Traditionally, cellular automata consist of a regular network with local uniform neighborhood dependency. However, in automata network (also called as cellular automata network), this uniform local neighborhood dependency is relaxed. Here, cellular automata rules allow a cell to have an arbitrary number of neighbors, and thus can be set to work on any given network topology [Tomassini et al. 2005; Tomassini 2006; Yang and Yang 2007; Darabos et al. 2007; Marr and Htt 2009]. Here, the rules can be either local or non-local [Boccaro and Roger 1994; Newman and Watts 1999]. “As the nonlocal rules are different from the

local rules, it is naturally expected that the nonlocal rules may lead to different behavior from conventional local rule-based CAs” [Yang and Yang 2007].

The earliest version of such non-uniformity in neighborhoods are found in [Jump and Kirtane 1974; Smith 1976]. However, from the 1990s, networks have been used as an important model for solving different complex problems; see for example [Adami 1995; Watts and Strogatz 1998]. In fact, after the work of [Watts and Strogatz 1998], researchers become more interested on automata networks. “Standard lattice cellular automata and random Boolean networks are extended to a wider class of generalized automata networks” [Tomassini 2006]. It is also shown that, automata networks with arbitrary topologies perform better than the regular lattice structures for the majority and synchronization problems [Tomassini et al. 2005; Darabos et al. 2007]. The work of [Cori et al. 1993], which is a pioneering work on ACAs and models concurrency and distributed systems, also uses automata network. In [Yang and Yang 2007], the authors have developed a new type of small-world cellular automata by combining local updating rules with a probability of long-range short-cuts to simulate the interactions and behavior of a complex system.

[Domosi and Nehaniv 2005] have investigated automata network as algebraic structures and developed their theory in line with other algebraic theories, such as semi-groups, groups, rings and fields. They have also shown a new method for the emulation of the behavior of any (synchronous) automata network by the corresponding asynchronous one. [Kayama and Imamura 2011] have shown the network representation of Game of Life, where the characteristics is like one of Wolfram’s class IV rules. In [Kayama 2012], the network derived from ECAs and five neighbor totalistic CA rules are further reviewed. However, the studies in this area are still at a very early stage.

5.3. Non-uniform CAs or Hybrid CAs

Among the above mentioned models, the most popular and studied model is *Hybrid CA* or *Non-uniform CA*, where the cells can use different local rules. The study of the non-uniform CA was started with [Pries et al. 1986], where the authors studied the group properties of 1-dimensional CAs under null and periodic boundary conditions. In that work, a special type of non-uniform CAs have been investigated, where the cells use Wolfram’s CA rules (see Table I). Since then, however, the major thrust of non-uniform CAs research has been on this class of CAs, see for example [Hortensius et al. 1989; Das et al. 1992; Chaudhuri et al. 1997; Das et al. 2004; Das and Sikdar 2009]. We dedicate the next section to survey this class of non-uniform CAs.

In recent years, the generalized definition of non-uniform CAs has been given [Cattaneo et al. 2009; Dennunzio et al. 2012b], where the cells may follow different rules with different neighborhood dependencies. Enrico and his colleagues have been investigating this class of non-uniform CAs, and they have identified various sub-classes of these CAs. Some basic global properties of non-uniform CAs, such as surjectivity, injectivity, equicontinuity, decidability, structural stability etc. have also been explored [Cattaneo et al. 2009; Dennunzio et al. 2012b; Dennunzio et al. 2014; Salo 2014].

6. NON-UNIFORM ECAS

Nowadays, research on non-uniform CAs has gained a popularity. The researchers have been exploring them from various directions, and proposing generalized definition of non-uniform CAs. However, since late 1980s until today, the primary focus of non-uniform CAs research has been on a special class of 1-dimensional CAs, where the cells follow Wolfram’s rules. The main reason of choosing this class of CA is two-fold – (1) Wolfram, in early 1980s, showed the efficacy of 3-neighborhood binary CAs in modeling physical systems and in producing complex global behavior, and (2) ease of implementing Wolfram’s CAs rules in hardware. We call these CAs as non-uniform ECAs, to differentiate them from others. Since the early

days, these CAs are explored targeting some hardware-related problems. Obviously, these CAs are fine.

Since different cells of a non-uniform ECA may use different rules, we need a rule vector $\mathcal{R} = \langle \mathcal{R}_0, \mathcal{R}_1, \dots, \mathcal{R}_{n-1} \rangle$ to define the CA. Here, cell i uses rule \mathcal{R}_i , and \mathcal{R}_i is presented as a decimal number as shown in Table I. Obviously, uniform or classical CAs are special cases of non-uniform CAs where $\mathcal{R}_0 = \mathcal{R}_1 = \dots = \mathcal{R}_{n-1}$. In this section, we will survey the research organized to understand the behavior of non-uniform ECAs. The CAs are mainly divided into two categories: additive/linear CAs and non-linear CAs.

Table III: Linear and complemented rules

Rule	Linear (With <i>XOR</i> logic)	Rule	Complement (With <i>XNOR</i> logic)
60:	$S_i(t+1) = S_{i-1}(t) \oplus S_i(t)$	195:	$S_i(t+1) = \overline{S_{i-1}(t) \oplus S_i(t)}$
90:	$S_i(t+1) = S_{i-1}(t) \oplus S_{i+1}(t)$	165:	$S_i(t+1) = \overline{S_{i-1}(t) \oplus S_{i+1}(t)}$
102:	$S_i(t+1) = S_i(t) \oplus S_{i+1}(t)$	153:	$S_i(t+1) = \overline{S_i(t) \oplus S_{i+1}(t)}$
150:	$S_i(t+1) = S_{i-1}(t) \oplus S_i(t) \oplus S_{i+1}(t)$	105:	$S_i(t+1) = \overline{S_{i-1}(t) \oplus S_i(t) \oplus S_{i+1}(t)}$
170:	$S_i(t+1) = S_{i+1}(t)$	85:	$S_i(t+1) = \overline{S_{i+1}(t)}$
204:	$S_i(t+1) = S_i(t)$	51:	$S_i(t+1) = \overline{S_i(t)}$
240:	$S_i(t+1) = S_{i-1}(t)$	15:	$S_i(t+1) = \overline{S_{i-1}(t)}$

6.1. Linear/additive CAs

A CA is linear if G , the global transition function of the CA, is linear. There are seven elementary CAs having rules 60, 90, 102, 150, 170, 204 and 240 which satisfy the linearity conditions (we have excluded rule 0 from the list, which also satisfies the conditions but a trivial one). These are linear ECAs, and the rules are linear rules. However, there is no additive ECAs other than linear ECAs. So in literature, the terms “linear CA” and “additive CA” are used interchangeably.

Linear/additive CAs are easily traceable by algebraic methods. In linear/additive CA, a next state function can be expressed by *XOR* logic (see Table III, which is reproduced directly from [Chaudhuri et al. 1997]). Since there are 7 linear/additive rules, the rule vector \mathcal{R} of a linear/additive CA is to be designed with only these 7 rules. A linear CA can also be expressed by characteristic matrix, T . The T is a tri-diagonal matrix, where all elements except the elements of main, upper and lower diagonals are always zero (see Section 3.2). In [Das et al. 1990; Das et al. 1992], the matrix algebraic tool has been used for analyzing state transition behavior of linear CA. From the matrix algebraic tool and characteristic polynomial, several interesting features of the CAs are derived.

Apart from the seven linear/additive rules, there are another seven rules which are complement of the seven. These rules are named as complemented rules, see Table III. The CAs with these complemented rules can also be characterized by matrix algebra. In fact, a CA that uses the fourteen rules, seven linear and seven complemented, can be efficiently characterized by algebraic tools, see for example [Chaudhuri et al. 1997; Ganguly et al. 2008]¹. However, for these CAs, we additionally need an *Inversion Vector* along with the characteristics matrix.

¹Some authors call the non-uniform CAs with linear/additive and complemented rules as additive CAs. The reason may be that, these CAs can be characterized using the tools of linear CAs. However, strictly speaking, these CAs are not additive, in general.

Linear/additive CAs are broadly classified as group CAs and non-group CAs, in literature. Next, we briefly discuss about them.

6.1.1. Group cellular automata. A CA is called a group CA if and only if the determinant $\det(T) = 1$. The naming of the subclass of linear CAs as group CAs comes from the fact that, these CAs form cyclic group “under the transformation of operation with T ” [Chaudhuri et al. 1997]. In a group CA, all the configurations are reachable from some other configurations of the CA. Hence, the group CAs are the reversible CAs.

In [Pries et al. 1986], it is stated that if \mathcal{R}_i is a group rule then its complement $\overline{\mathcal{R}_i}$ (that is, $255 - \mathcal{R}_i$) is also a group rule, which is proved in [Das 1990]. A subclass of group CAs is maximal length CAs, in which all non-zero configurations lie in the same cycle. These CAs produce high quality pseudo-random patterns, and are utilized in electronic circuit testing [Hortensius et al. 1989; Bardell 1990; Serra et al. 1990].

In case of maximal length CAs, the characteristics polynomial is primitive. [Cattell and Muzio 1996] provides a scheme of synthesizing a maximal length CA from a given primitive polynomial over $GF(2)$. Rules 90 and 150 are used to construct such CAs, and no single rule can produce a maximal length CA. In [Cattell and Zhang 1995], the maximal length CAs are synthesized up to size 500, where one or two cells follow rule 150 and the rest follow rule 90. However, for a periodic boundary CA, the characteristic polynomial is factorizable, therefore, there exists no maximal length CA under periodic boundary condition [Nandi and Chaudhuri 1996].

6.1.2. Non-group cellular automata. These are irreversible linear/additive CAs. Here, the characteristics matrix T is singular, where the T of a group CA is non-singular. Non-group CAs are explored in different areas, see for example [Bhattacharjee et al. 1995; Chakraborty et al. 1996; Chattopadhyay et al. 2000; Roncken et al. 2000]. Any non-group CA is characterized by the following terms -

- *attractors*: cyclic states form attractors. If an attractor contains only a single state, it is said to be in graveyard state or a point state attractor or fixed point.
- α -*basin* or α -*tree*: the set of state(s) rooted at any attractor state α , is termed as α -basin or α -tree.
- *depth* or *height* of a CA represents the “number of steps required to reach the nearest cyclic state from a non-reachable state” [Shaw et al. 2004b].

Some vital findings about number of predecessors of the all-zero configuration and depth of CA are reported in [Chaudhuri et al. 1997]. A fraction of all reachable/non-reachable configurations of a uniform CA have been identified by [Martin et al. 1984]. However, in general for any additive CA (uniform/hybrid), the fraction of reachable/non-reachable configurations can be numerated from the knowledge of the number of predecessors of a reachable state [Chaudhuri et al. 1997]. Latter, an efficient scheme has been developed to identify and count reachable and non-reachable configurations of a (uniform/non-uniform) CA, which can be linear and as well as non-linear, in [Das and Sikdar 2008].

Some fascinating classes of non-group CAs are also explored, like - multiple attractor CAs (MACAs) [Chattopadhyay et al. 2000; Maji et al. 2003b], depth-1* CAs ($D1 * CAs$) [Chowdhury 1994] and single attractor CAs (SACAs) [Das et al. 1992]. These CAs have been employed in a broad range of purposes like hashing [Ganguly et al. 2000], classification [Ganguly et al. 2002a; Maji et al. 2003b], designing easy and fully testable FSM [Chowdhury et al. 1993] etc.

6.2. Non-Linear CAs

Initially, non-uniform non-linear CAs have not been widely explored, due to absence of proper characterization tool for non-linear CAs. In the past, several attempts have been made to study the characteristics of uniform CAs (linear or nonlinear) qualitatively and

quantitatively in terms of parameters, such as λ parameter [Langton 1990], Z parameter [Wuensche 1994; Wuensche 1998] etc. de Bruijn graph is also considered as a characterization tool of CAs (linear/nonlinear) [Sutner 1991; Provillard et al. 2011]. However, they were not appropriate to study non-uniform non-linear CAs.

A new characterization tool, named *reachability tree* (see Section 3.3), has been proposed in [Das et al. 2004; Das 2007] to characterize any linear/nonlinear uniform/non-uniform CAs. Utilization of this tool to characterize non-linear CAs are discussed in [Das and Sikdar 2006; Das et al. 2008; Das and Sikdar 2008; Das 2007]. The efficacy of reachability tree to identify and synthesize reversible CAs as well as identify attractor sets is proved in [Das and Sikdar 2006; Das et al. 2008]. In [Naskar et al. 2012a], reachability tree has been used to distinguish cyclic/acyclic configurations from the configuration space of non-linear CA. In [Naskar et al. 2012b; Naskar et al. 2014], 1-dimensional non-linear CA with point state attractors are also explored using reachability tree. Recently, rule vector graph (RVG) is also developed for studying invertibility of 3-neighborhood CA with null boundary condition [Maiti et al. 2010; Ghosh et al. 2011].

Non-linear CAs are proved to be efficient in various application fields, like VLSI design and test [Das 2007], pattern recognition and classification [Maji 2005; Das et al. 2009b; Das et al. 2009a] etc. Further, identifying reversible CAs, synthesis of reversible CAs along with designing pseudo-random pattern generator (PRPG) around reversible non-linear CAs are reported in [Das et al. 2003; Das et al. 2004; Das et al. 2005].

In [Ganguly et al. 2002b], the concept of *GMACA* (generalized MACA) has been introduced for non-linear CAs. The efficiencies of *MACA* and *GMACA* have been compared and reported in [Ganguly 2004] with respect to pattern recognition. In [Maji 2005], both linear and non-linear CAs are used for designing a pattern classifier.

Fuzzy CA, a natural extension of boolean CA, is analyzed and synthesized using matrix algebraic tool in [Maji and Chaudhuri 2005; Maji and Chaudhuri 2004]. This CA is also used to design pattern classifier [Maji and Chaudhuri 2007; Maji 2005].

7. CELLULAR AUTOMATA AS TECHNOLOGY

Technology refers to the collection of techniques, methods or processes used to provide some service or solutions to problems, or in the accomplishment of an objective, such as scientific invigilation. The CAs have been historically used as a method of biological and physical systems, and utilized to theoretically study such systems. Since late 1980s, however, the CAs have been started to be used as solutions to many real-life problems. In this section, we survey some of such solutions.

7.1. Electronic circuit design

The CAs, particularly non-uniform ECAs, have received their popularity as technology in the era of VLSI. The simplicity, modularity and cascability of CA have enticed the researchers of VLSI domain. Some of these areas are briefly described here.

7.1.1. Early phase developments: CAs based machines, CAMs (CA Machines), having high degree of parallelism have been developed in [Toffoli and Margolus 1987], which are ideally suited for simulation of complex systems. Even before the introduction of CAM, CAs have been utilized as parallel multipliers [Atrubin 1965; Cole 1969], parallel processing computers [Manning 1977], prime number sieves [Fischer 1965], and sorting machine [Nishio 1981]. Design of fault-tolerant computing machine [Nishio and Kobuchi 1975] and nanometer-scale classical computer [Benjamin and Johnson 1997] are also commendable works.

7.1.2. VLSI Design and Test: Hortensius [Hortensius et al. 1989; Hortensius et al. 1989] proposed hybrid CA based Pseudo Random Pattern Generator (PRPG) for built-in self-test (BIST) in VLSI circuits. Some of the major contributions in the research of PRPGs based on CAs are reported in [Das 1990; Tsalides et al. 1991; Chowdhury 1994; Das et al.

2004; Das et al. 2005]. The CAs are also proposed as a framework for BIST structures [Tsalides 1990; Das 1990; Chowdhury 1994; Das 2007; Chakraborty and Chowdhury 2009] and as a deterministic test pattern generator [Albicki and Khare 1987; Das and Chaudhuri 1989; Das 1990; Das 2007]. Utilizing the scalability of CAs, a test solution for multi-core chips has been proposed in [Das and Sikdar 2010]. While testing a CUT (Circuit-Under-Test) with pseudo-random patterns, a set of patterns may be prohibited to the CUT which may adversely affect the circuit. CAs based solutions to this problem have been proposed that can generate pseudo-random-patterns without PPS (Prohibited Pattern Set) [Ganguly et al. 2002d; Das et al. 2003]. Finally, a universal test pattern generator, termed as the *UBIST* (Universal BIST) has been reported [Das et al. 2003; Das and Chaudhuri 1993]. This “*UBIST* can generate any one of the four kinds of test patterns (i) pseudo-random, (ii) pseudo-exhaustive, (iii) pseudo-random without PPS (Prohibited Pattern Set), and (iv) deterministic” [Das 2007].

7.1.3. Synthesis of Finite-State Machine (FSM): [Mitra et al. 1991; Misra et al. 1992] depicted the design of testable FSM with CAs. In [Chowdhury et al. 1993; Chakraborty et al. 1996], some fascinating properties of a non-group CA and its dual and their relationship are reported. These papers have also explored a particular class of non-group CAs, named as *D1**CAs, which have been recommended “as an ideal test machine which can be efficiently embedded in a finite state machine to enhance the testability of the synthesized design” [Chakraborty et al. 1996].

7.1.4. Security and others. A good number of works are reported in literature that deal with CAs based encryption, cryptography and authentication techniques; see for example [Nandi et al. 1994; Bao 2004; Seredynski et al. 2004; Seredynski and Bouvry 2005; Das and Chowdhury 2011; Das and Chowdhury 2013]. Other notable works, related to electronic circuit design are CAs based error correcting codes [Chowdhury et al. 1994; Paul and Chowdhury 2000], signature analysis [Hortensius et al. 1990; Das et al. 1990], etc. For more discussion, see [Chaudhuri et al. 1997].

7.2. Computer vision and machine intelligence

A group of CA researchers has also explored CAs in the fields of image processing, pattern recognition etc. These fields are roughly named as computer vision and machine intelligence.

7.2.1. Image processing: CAs, specially 2-dimensional CAs, are used for image processing. They can address all the significant image processing works like translation, zooming, rotation, segmentation, thinning, compression, edge detection and noise reduction etc [Rosin 2006; Rosin 2010; Kazar and Slatnia 2011]. [Khan 1998] has showed that using hybrid CAs, it is possible to rotate any image through an arbitrary angle. In [Paul et al. 1999], a new GF (2) CA based transform coding scheme for gray level and color still images has been proposed. “This scheme is computationally simple and achieves superior reconstructed image quality at higher compression ratios” [Paul et al. 1999]. Some works regarding edge detection and noise reduction are reported in [Wongthanavasut and Sadananda 2003; Sadeghi et al. 2012].

7.2.2. Pattern recognition: CAs have been a popular tool for pattern recognition and classification since long [Jen 1986; Raghavan 1993; Chaudhuri et al. 1997]. A particular class of CA - *MACA* (multiple attractor CA), can act as a natural classifier [Ganguly 2004]. The correlation between *MACA* and *Hamming Hash Family (HHF)* is shown in [Ganguly et al. 2000]. Hamming hash family has inherent capacity to address classification task. This basic framework of linear CA is extended to a special class of non-linear CA, known as generalized *MACA* (*GMACA*) and utilized for modeling the associative memory [Maji et al. 2003a; Maji 2005]. In [Maji and Chaudhuri 2004], fuzzy CAs have been explored as an efficient pattern classifier. Non-linear CAs based pattern classifiers are further developed in [Das

et al. 2009a; Das et al. 2009b]. Recently, an asynchronous CA based pattern classifier is reported in [Sethi et al. 2016].

7.2.3. Compression and others: In [Bhattacharjee et al. 1995], some methods are proposed to perform text compression using CA as a technology. In [Lafe 1997], cellular automata transforms are proposed for digital image compression and data encryption. It is shown that, CAs can generate orthogonal, semi-orthogonal, bi-orthogonal and non-orthogonal bases. CA based transforms are also presented in [Paul et al. 2000; Shaw et al. 2004b; Shaw et al. 2006] for developing efficient schemes of image and document compression. Some other related works are reported in [Lafe 2002; Ye and Li 2008]. A technology, called *Encompression*, where encryption and compression are married, has been reported in [Shaw et al. 2003; Shaw et al. 2004a].

7.3. Medical science

Since the seminal work of von Neumann in 1950, CA has attracted attention of computer scientists and biologists as an excellent tool to model self-replicating system. The reason of choosing CA for biological modeling is – (1) it is fast and easily implementable and (2) the visual result of the simulation provides remarkable resemblance with the original experiment. In [Burks and Farmer 1984], the research on modeling the evolution and role of DNA sequences within the framework of CA has been initiated. [Ermentrout and Edelstein-Keshet 1993] has reviewed a number of biologically motivated CA, such as deterministic or Eurlian automata, lattice gas models and solidification models, heuristic CA etc. and shown the effectiveness of CA in understanding a physical process. In [Mitra et al. 1996], a pioneering work on modeling amino acid using $GF(2^2)$ CA, referred as amino acid CA (AACA) is reported. Some other interesting works are recorded in [dos Santos and Coutinho 2001; Moreira and Deutsch 2002; Xiao et al. 2006; Santos et al. 2013].

CGI detection in DNA sequence is a known problem in biological sequence analysis. [Ghosh et al. 2007] has solved this problem. In [Ghosh et al. 2010], a noteworthy class of CAs, known as *Equal Length Cycle CA*, has been proposed to predict and classify the enzymes. In [Ghosh et al. 2012], another notable class of CAs, termed as *restricted 5-neighborhood CA* (*R5NCA*) is introduced to predict protein structure, and developed a Protein modelling CA Machine (*PCAM*). Here, a *R5NCA* rule is used to model an amino acid of a protein chain. This PCAM has been used for synthesis of protein structure using an organized knowledge base [Ghosh et al. 2014].

8. CONCLUSION

Cellular automaton has gone through a long journey from the early period of von Neumann to elementary form of Wolfram, and to the modern trends of research using this simple yet beautiful model. In this survey, various milestones of development regarding CAs are briefly depicted, such as the variety of types, different characterization tools, global behaviors and non-uniformity. However, in all the alternatives, some basic characteristics are modified with respect to the initial model of cellular automaton proposed by von Neumann. Among these, non-uniformity in CAs, specially in elementary CAs, has raised magnificent interest among the researchers of the last two decades. These CAs have been used in developing solutions to many real life problems.

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